The Non-Convexity Issues of a Limited-Commitment Economy

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Abstract

After reviewing basic self-enforcing labour contracts models, we expose how self-enforcing labour market theory can help explain some important dynamic properties of key macroeconomic variables. The idea of adapting the microeconomic theory of Thomas and Worrall (1988) to address the issue of macroeconomic dynamics was first proposed in Calmès (1999, 2003). He details how self-enforcing labour contracts improve the way macroeconomic models account for the response of the economy to external shocks. The introduction of a state-dependent outside opportunity for the manager is the first step in generalizing the theory (Calmès 2007, Thomas and Worrall 2007). In this paper we discuss the next step, the endogenization of capital. Although desirable, this task is not straightforward as the contract set might no longer be compact in this case. Relatedly, we also discuss the introduction of a third agent (the financial intermediary) in the model. We also analyse the link between stationarity and set convexity when incorporating growth in the model. A stochastic trend may be considered but then the non-convexity issue arises again. The aggregation of heterogeneous individual contracts can also lead to the same problem.

Keywords: Internal propagation mechanisms; Real business cycle; Self-enforcing contract; Risk-sharing hypothesis; Non-convexity.

JEL: E12, E49, J30, J31, J41.

Résumé

Après avoir passé en revue les principaux modèles de la théorie des contrats de travail auto-exécutoires, nous montrons comment cette théorie peut contribuer à mieux expliquer plusieurs propriétés dynamiques des variables macro-économiques clefs. Calmès (1999, 2003) fut le premier à proposer la transposition de la théorie microéconomique de Thomas et Worrall (1988) *à l'analyse de la dynamique macroéconomique. Il vérifie que l'introduction de contrats auto-exécutoires dans un modèle macroéconomique peut améliorer la réaction des agrégats aux chocs externes. L'ajout d'une opportunité externe pour l'entrepreneur liée aux états de la nature constitue la première étape pour généraliser la théorie des contrats de travail auto-exécutoires (Calmès 2007, Thomas et Worrall 2007). Dans cet article, nous discutons la deuxième étape, soit l'endogénéization du capital. Bien que désirable, celle-ci ne va pas sans difficultés puisque l'ensemble des contrats pourrait ne plus être compact dans pareil cas. Parallèlement, nous envisageons la présence d'un troisième agent (un intermédiaire financier) dans le modèle. Nous étudions également le lien entre la stationnarité et la convexité de l'ensemble des contrats lors de l'incorporation de la croissance dans le modèle. À cet effet, un trend stochastique peut être ajouté mais alors le problème de la non-convexité surgit de nouveau. L'agrégation de contrats individuels hétérogènes peut donner jour au même problème.*

Mots-clefs : Mécanismes de propagation interne; Cycle économique réel (RBC); Contrat auto-exécutoire; Hypothèse du partage du risque; Non-convexité.

JEL : E12, E49, J30, J31, J41.

1. Introduction

The dynamic contract theory has evolved a long way since the introduction of implicit contracts by Bailey (1974), Gordon (1974) and Azariadis (1975). At first, it was used to address the issue of wage stickiness, the flexible wage model being unable to account for key labour market properties (Beaudry and DiNardo 1995). With the endogenization of wage rigidity, implicit labour contract models provide a more satisfying explanation of the relationship between key labour market variables like average productivity, hours worked and real wage. In these models, the fisherian equality between the wage and the marginal product of labour no longer holds, and therefore, the implied dynamics of these models is very different from the one associated to the flexible wage models. However, the implicit labour contract models were not well adapted to the study of macroeconomic dynamics.

Self-enforcing labour contract models may be viewed as a second generation of implicit labour contract models. They generalize the theory with the introduction of commitment problems, the principal and the agent being unable to commit to their agreements. To be self-enforcing, the contracts must give to both parties more than what outside opportunities offer.

In a partial equilibrium, microeconomic setting, Thomas and Worrall (1988) were the first to suggest that the risk-sharing hypothesis could be used to describe the *dynamics* of labour relationships. Boldrin and Hovarth (1995) use this theory to study the dynamics of employment, consumption, productivity, hours worked and real wages. They focus on a very restricted form of dynamics, i.e. contemporaneous (instantaneous) dynamics. In other words, their results mainly concern contemporaneous macroeconomic dynamics. Incidentally, most models featuring dynamic contracts only consider one-period contracts (Boldrin and Hovarth 1995, Danthine and Donaldson 1992).

Using long term contracts instead, Calmès (1999) deals with the pervasive impact of the income effect which strongly dampens the substitution effect in standard RBC models. Relying on the risk-sharing hypothesis helps weaken the income effect because the insurance provided by long term contracts tends to smooth the marginal utility of consumption intertemporally *and* intratemporally, with the consequence of more prolonged comovements of consumption and hours (Calmès 2007).

There is another dimension in which dynamics is quite fundamental and yet often neglected by researchers applying dynamic contract theory. We refer here again to the *intertemporal* dimension of dynamics. In macroeconomics, the internal propagation mechanisms incorporated in the standard models must help replicate the actual impulse-response functions of key macroeconomic aggregates. The propagation of shocks described by these models should tend to match the empirical stylized facts. While progress has been great in this area, especially with models featuring nominal and real rigidities, most models still fail to account for the observed dynamic properties of impulse-response functions of output (Cogley and Nason 1995). The amplitude of the cycles generated by the RBC models is generally too low and the artificial cycles die too quickly. The simulated volatility of macroeconomic aggregates is also too low. These contradictions between the behaviour of macroeconomic time series and their theoretical counterparts pose a challenging puzzle.

In this respect, self-enforcing labour contract models seem to offer a promising propagation mechanism. Calmès (2007) transposes the methodology of Thomas and Worrall (1988) in a macroeconomic setting to study the intertemporal response of a limited-commitment economy to aggregate shocks. He shows a rich (intertemporal) dynamics obtains, the results supporting the conjecture that the lack of dynamics generally encountered in RBC models stems from the embedded excessive strenght of the dampening income effect[1].

The organisation of this paper is as follows. First, we present the core of Calmès' thesis (2007), which links the importance of the comovements between macroeconomic aggregates to the weakness of the income effect. Second, we discuss the respective contributions of Thomas and Worrall (1988, 2007) and Calmès (2007). We show how self-enforcing contracts help explain macroeconomic dynamics, in the context of the two-sided limited-commitment economy suggested in Calmès (1999). We then detail the non-convexity issues researchers face when generalizing the self-enforcing labour market theory. Among others, topics include: the aggregation of individual, heterogeneous

[1] i.e. the strength of the income effect dampens economic fluctuations in standard macroeconomic models.

contracts, the introduction of economic growth, variable contract duration, and endogenous capital.

2. Income Effects and Macroeconomic Dynamics

In this section, we highlight the peculiarities of the macroeconomic dynamic properties of the limited-commitment models by comparing two extreme settings: a flexible wage economy and a full-commitment one. We show that the dynamic properties derived from these two paradigms are quite different, mostly because of the strong influence of the income effect[2] in the former model and its quasi-absence in the latter. In the first part of this section, we study the relationships between three key macroeconomic variables related to the labour market - wage, hours worked and productivity - in relation to the substitution and the income effects. Then, we discuss how the risk sharing hypothesis, by eliminating the income effect, generates a richer dynamics within the full-commitment economy.

Let us first define our two macroeconomic frameworks. The spot market economy, associated to autarky, also called the flexible wage model, is characterized by the absence of risk sharing. At the other extreme, in the full-commitment economy, risk sharing is provided by the implicit labour contracts. In this kind of contracts, the firm manager and the workers fully commit to never renege. Suffice to say here that these contracts give way to a perfect smoothing of consumption[3] across states and time. These two kinds of models have very different macroeconomic dynamic properties because of the difference in the way the income effect plays its role in each of them.

In the spot market model, the income effect has a strong influence on the equilibrium. Following a positive technological shock – i.e. a shock which increases productivity – the substitution effect produces an increase in hours worked but this

[2] Note here that the labour contracts literature does not limit the income effect to a change in wages and is often vague on the nature of this effect. Generally, the income effect is defined as the negative relationship between wage and hours worked, an increase in wage giving way to a decrease in hours worked. But an income effect might also be generalized to an endowment one. For instance, an increase in marginal productivity of labour gives way to an income effect. This effect is obvious in the spot market model because wage is equal to the marginal productivity of labour. But if this is not the case, an increase in productivity, by increasing the agents endowments, actually leads to an income effect. In this respect, income (or substitution) effects are often introduced with technological shocks causing changes in productivity.

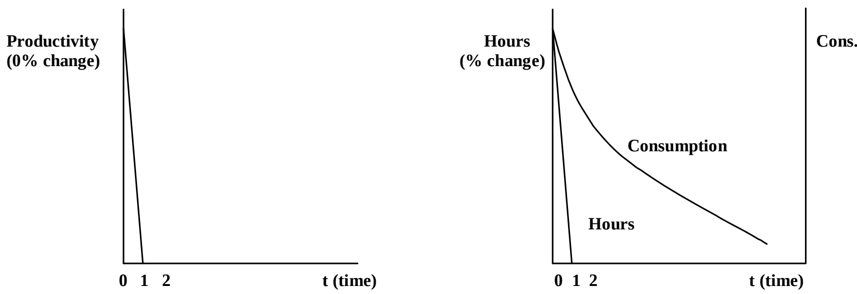
[3] More precisely, marginal utility of consumption is smoothed across states and time.

change is completely neutralized by the income effect, leading to a correlation at odds with the data. As wage is equal to marginal productivity of labour in this model, in the long run, the overall correlation between wages and hours worked is either null or negative. In any case, the effects of shocks die quickly. Thus shocks are not amplified and are weakly propagated, at best.

Instead, in the full-commitment economy, the income effect is totally mitigated. This neutralization is due to two consequences of the risk sharing hypothesis: i) the insurance incorporated in the workers earnings dissociates wage from the marginal productivity of labour. Wage is thus no longer equal to the marginal productivity of labour in this model, it is sticky and generally governed by a non-linear updating rule. According to Rosen (1985), it is this non-linear pricing mechanism which contributes to eliminate the income effect. Hence, the movements in the marginal productivity of labour do not give rise to the usual income effect; ii) full risk-sharing also results in contractual transfers perfectly smoothing consumption across states and time. Consequently, there is no income effect in this kind of setting (Hart and Holmström 1987), as workers are no longer preoccupied by income variations.

Since the income effect is eliminated in the full-commitment economy, hours worked are essentially governed by the movements in productivity. A positive technological shock entails an increase in hours worked because the substitution effect is the only one at play here. And, as earnings are fixed, this shock leads to a wage decrease, in contradiction to the facts, and there is thus a negative correlation between wage and hours worked in this model.

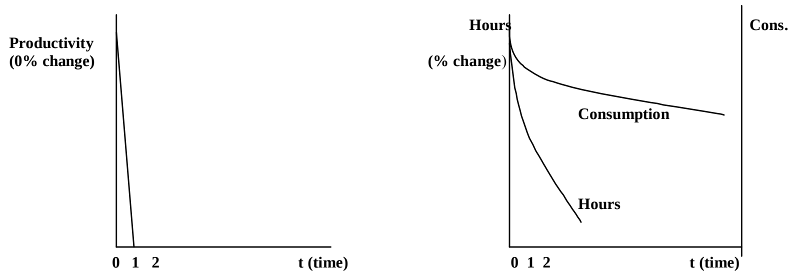
Figure 1 *Transitory productivity shock in the flexible wage economy*[4]



In the flexible wage model, after a positive transitory technological shock, consumption and hours worked increase but they diverge quickly, the income effect knocking down the rise in hours, as illustrated in figure 1. Following the shock, the substitution effect gives way to a rise in both hours worked and consumption, but the hours worked return rapidly to their initial level. However, the subsequent decrease in consumption growth is smoothed, the decreasing marginal utility of consumption leading consumers to postpone a portion of the additional consumption associated to the shock in the future (King and Rebelo 1999)[5]. The spot market model has also the tendency of understating the volatility of employment and overstating the volatility of the real wage. Consequently, this model cannot account for the persistence in the movements of the aggregates (Cogley and Nason 1995). The intertemporal comovement between consumption and hours worked is supposed to be long lived according to the empirical evidence. Wage also lacks persistence in the spot market model.

[4] This figure is inspired from : King and Rebelo (1999).
[5] On the dynamic properties of the flexible wage model, see also: King, Plosser and Rebelo (1988a, b) and Rotemberg and Woodford (1996).

Figure 2 *Transitory productivity shock in the full-commitment economy*



By contrast, the key advantage of the full-commitment model stems from the fact that the income effect is completely eliminated, which enables the substitution effect to amplify and propagate the shocks. Figure 2 illustrates the behaviour of consumption and hours worked following a positive transitory productivity shock. The substitution effect, leading to an increase in hours worked, and this effect not being countered by the income effect, translates into more prolonged comovements between consumption and hours. This is why the dynamics delivered in such a framework is much more powerful than in the standard one. In the standard framework, it is precisely the lack of a prolonged comovement between consumption and hours which dampens the internal propagation of shocks (Calmès 2002).

In the following section, we study the limited-commitment economy, a hybrid between the spot market and the full-commitment economies. The partial risk-sharing inherent to the limited-commitment economy does not eliminate the income effect entirely but contributes to dampen its impact. Thus, the limited-commitment economy displays a richer dynamics than the spot market model, and a less trivial dynamics than the full-commitment case, where the substitution effect is the only one at play. Although less pronounced than in the full-commitment economy, the macroeconomic movements of aggregate variables are more consistent with the data in the limited-commitment economy than in the full-commitment one, where earnings are quite fixed, which constitutes an extreme configuration.

3. Limited-Commitment Models and the Dynamics Puzzle

3.1 Thomas and Worrall Models and Macroeconomic Dynamics

From the perspective of macroeconomic theory, a major advantage of the model proposed by Thomas and Worrall (1988) is that it provides a natural rationale for real wages rigidity and persistence. The most important contribution of the authors is to derive a real wage updating rule more in accordance with the properties observed in the data than the one associated to the flexible wage model. Interestingly, this updating rule entails a nonlinear pricing rule which can be linked to option theory. Indeed, according to Rosen (1985, p. 5): "The opportune substitution of work effort toward more productive states has a value similar to that of an option: that less work is called for in the less favourable states serves to truncate the lower tail of the θ[6] distribution". Hence, this connection between option theory and limited-commitment models also offers the possibility of a complete reformulation of these models using options.

As a matter of fact, Thomas and Worrall (1988) compare the entrepreneur featured in their limited-commitment model to a risk-neutral principal providing insurance to a risk-averse agent facing an i.i.d. income stream. It is possible to consider the spot wage a random income and the contract a net income transferred from the insurer. This comparison to insurance contracting relates to option pricing directly. In self-enforcing labour contracts, it is the insurance motive which partially dissociates real wage from marginal productivity. In the real option theory, it is the real options embedded in an investment project which operate the dissociation between real wage and marginal productivity (Dixit and Pindyck 1994). Hence labour insurance works as a put option. More precisely, the outside opportunities of the agents can be modelled as options.

In Thomas and Worrall (1988), there is an infinite sequence of dates and a finite sequence of states $s_t \in \{1,2,...,S\}, S \geq 2$, two types of agents, entrepreneurs and workers, and contracts are self-enforcing in the sense that no firm's agent has an incentive to renege. Workers are infinitely lived, risk-averse and have an identical per period state independent utility function $u = u(\omega)$, where ω stands for the wage provided by the contract. *u(.)* is twice differentiable and strictly concave. All managers are infinitely lived

[6] θ being a random variable accounting for technological uncertainty in the model.

and risk-neutral. Both agents discount the future with a common factor $\alpha \in (0,1)$. In this model, each state is identified solely by the spot market wage *w(s)*, with probability of state *s* denoted by *p(s)*. Note that at this stage of development of the theory there is no production function in the model.

It is assumed that the entrepreneurs have complete access to capital markets so they can diversify the idiosyncratic risks associated to the operations of the firm they manage. On the other hand, workers consume all their earnings and have no access to capital markets. Being risk-averse, they prefer stable earnings to fluctuating ones. The state that occurs at time *t* is denoted by s_t. The history of states up to time *t* is designated by $h_t = (s_1, s_2, \ldots s_t)$. A contract, δ, is a sequence of functions $(\omega(h_t))_{t=1}^{\infty}$, where $(\omega(h_t))$ is the wage paid if the history is h_t. Note the different notation for the spot market wage *w* and the wage ω offered by the firm. We want to find the set of contracts which is Pareto efficient. A contract is efficient if there exists no other self-enforcing contract offering both parties at least the same expected utility and strictly more to one of the firm's agent. In other words, the set of Pareto-efficient contracts maximizes the present value of the firm expected cash flows for a given level of the worker's present value of expected utility. Hence, the entire Pareto frontier can be traced by varying the level of the worker's present value of expected utility. The dynamic program used to compute the Pareto efficient set of contracts includes two self-enforcing constraints (SEC), one for each representative agent[7]. To be self-enforcing for an agent, a contract must provide him as much payoff as he could get from his outside opportunity. Note that in Thomas and Worrall (1988), this outside opportunity is the spot labour market for both the worker and the entrepreneur, and this opportunity is exogenous. A worker reneging then works on the spot market forever. An entrepreneur can also renege, firing the employees and hiring on the spot market.

[7] One can argue that there is only one representative agent, the firm, composed of two types, an employee and a manager and described by their strategic interactions. In that respect, Calmès (2002) compares this “agent moyen” to Robinson Crusoé and Friday. This representative agent, anticipating upcoming shocks, has to decide his intertemporal strategy in an uncertain environment. In other words, he must determine his consumption, savings, working and production plans for all possible states and future dates.

Contrary to what happens in the full-commitment case, where the entrepreneur completely insures the earnings of the worker, in a limited-commitment environment risk-sharing is only partial.

For each contract δ and any history h_t, the net gains accrued to the worker are given by the following equation

$$U(\delta;h_t)=u(\omega(h_t))-u(w(s_t))+E\sum_{\tau=t+1}^{\infty}\alpha^{\tau-t}\{u(\omega(h_\tau))-u(w(s_\tau))\}$$

where the worker's SEC is defined as:

$$U(\delta;h_t)\geq 0$$

The net future benefit that the entrepreneur gets from the contract at time t after history h_t is given by the following equation:

$$\Pi(\delta;h_t)=w(s_t)-\omega(h_t)+E\sum_{\tau=t+1}^{\infty}\alpha^{\tau-t}\{w(s_\tau)-\omega(h_\tau)\}$$

where SEC for the entrepreneur is

$$\Pi(\delta;h_t)\geq 0$$

If $w_t(s_t)<\omega(h_t)$, there is an incentive for the entrepreneur to hire employees on the spot market. To avoid reneging, the expected wages he offers has to be lower than the expected wages prevailing on the spot market. Also, if $u(\omega(h_t))<u(w(s_t))$, then the worker has a short-term incentive to renege because he would be better off with the spot market allocation. This incentive must be compensated by the long-term benefit from compliance to the relationship. More precisely, the Holmström backloading principle prevents reneging. To understand this, we can illustrate the situation in the following setup due to Holmström (1983). Let us assume for a moment a two period contract. In such a contract, workers pay their insurance premium in advance so that they receive initially a wage below their productivity to be compensated later with wages above productivity. The point is that workers are not hired by firms paying at the marginal product rate in the first period, as would be the case on the spot market. Yet, to compensate, worker reliability is rewarded by seniority rule. Worker's compensation is thus backloaded, which generates a repeated relationship. Wages are lower in the first period because the firm must offer the worker at least the spot market wage in the second period. To summarize, the entrepreneur

provides earnings stability to the worker in exchange of lower expected wages. This is the insurance device we also find in usual self-enforcing models.

At this stage, it is important to note that, if agents could renege and recontract next period without penalty, "no non-trivial self-enforcing contract" would exist according to Thomas and Worrall (1988, p. 542). For them, frictions must thus be introduced in self-enforcing labour contract models in order to avoid this case. Thomas and Worrall (1988) propose that any reneging agent is observed by everyone else. Reneging must thus harm reputation. Further, they also assume an infinite contract duration and suppose that once an agent has reneged, he must trade on the spot market from then onwards. These strong assumptions are a priori necessary precautions to ensure that, in equilibrium, all contracts are non trivial solutions. But these features are somewhat unsatisfactory. We might even question how they might really help. Instead, as discussed below, Thomas and Worrall (2007) introduce explicit reneging costs in their model, which, as we explain in the next section, is a much more convincing device to prevent reneging in this kind of setting.

To compute the Pareto efficient set of contracts, Thomas and Worrall (1988) define the expected utility level as an endogenous state variable[8]. With this method, finding the Pareto frontier amounts to solving a regular dynamic programming problem. For an history (h_{t-1}, s) and a feasible value of the worker's expected utility U_s^t, the Pareto frontier is given by:

$$f_s\left(U_s^t\right) = \sup_{\delta \in \Lambda\left(h_{t-1}, s\right)} \left\{ \Pi\left(\delta; \left(h_{t-1}, s\right)\right) \middle| \, U\left(\delta; \left(h_{t-1}, s\right)\right) \geq U_s^t \right\}$$

Since an efficient contract cannot be Pareto dominated after any history, it follows that the corresponding dynamic program can be written as[9]

$$f_s\left(U_s^t\right) = \sup_{\omega(s), \left(U_q^{t+1}\right)_{q=1}^S} w(s) - \omega(s) + \alpha E f_q\left(U_q^{t+1}\right)$$

s.t.

[8] Note that this type of variable could be interpreted as the second jump variable identified by Wen (2005) as necessary to properly account for the dynamic properties of the state space associated to the canonical RBC model.

[9] The binding budget constraint of the worker, i.e. that he consumes all his earnings, is already incorporated in his utility function.

the two SEC:

$$U_q^{t+1} \geq 0; f_q\left(U_q^{t+1}\right) \geq 0, \ for\ q = 1,2,...,S$$

and the feasibility constraint, with the expected utility level *u(.)* of the worker such that

$$u(\omega(s)) - u(w(s)) + \alpha E U_q^{t+1} \geq U_s^t$$

The last constraint ensures that the utility level which the worker gets from the contract is at least equal to his expected utility level at time *t*. It can be shown that this program is concave. In any case, Thomas and Worrall (2007) also introduce a simpler method for finding the Pareto efficient set of contracts, without resorting to a dynamic programming argument but using instead local variational arguments − the conventional method for the Pareto frontier computation. Indeed, they explain that using local variational arguments instead of dynamic programming avoids the need to establish a number of technical properties of the value function, including the restrictions of twice differentiability. The entire Pareto frontier is thus traced by varying the level of the worker's expected discounted utility.

According to this alternative formulation, the program to retrieve the Pareto efficient frontier becomes

$$\sup_{\omega(h_t)_{t=1}^T} \Pi(\delta; h)$$

s.t.

the two SEC of the worker and the entrepreneur

$$U(\delta; h) \geq 0$$

$$\Pi(\delta; h) \geq 0$$

and the promised utility level of the worker

$$U(\delta;h) \geq \overline{U}$$

The last constraint defines the level of utility the worker receives when nature has played and a particular state realizes. The term $\overline{U}$ measures how much utility the worker gets from the relationship, and as this level is varied across feasible values, all efficient contracts are traced out. An important first-order condition associated to a Pareto efficient contract is the Arrow-Borch-Wilson formula governing optimal risk-sharing[10]. This condition states that risk-sharing is optimal when the marginal rate of substitution of consumption between two different states or time periods is the same for both agents engaged in the contract. Since one agent is a firm here, it is more appropriate to refer to "the marginal rate of substitution of profit" for this agent. Actually, the value function might be defined in terms of the utility of profit instead of rough profit, if the firm is risk-averse instead of being risk-neutral as assumed in the model (Cahuc and Zylberberg 2001)

This problem gives rise to a simple updating rule for the real wage. For any history (h_{t-1}, s), Thomas and Worrall (1988) found that the wage associated to an efficient contract, $\omega(h_{t-1},s)$, is included in a closed non-empty interval $(\underline{\omega}_s, \overline{\omega}_s)$. If it is equal to $\underline{\omega}_s$, the worker gets no gain from the contract, so $U_s^t = 0$. If, on the other hand, $\omega(h_{t-1}, s) = \overline{\omega}$, the manager gets no gain , so $f_s(\overline{U}_s) = 0$. Contract wages vary over time according to a simple pattern, keeping wages fixed if possible but changing them by the smallest possible amount otherwise. Note also that if the firm's SEC is slack, wages cannot fall. On the other hand, if the worker's SEC is slack, wages cannot raise.

3.1.1 Towards a Macroeconomic Formulation of a Limited-Commitment Model

Discussing variable hours, Calmès (1999, 2003) describes a simple wage equation much in the spirit of the Thomas and Worrall original formulation, resorting to their methodology to account for the rigidity of real wage. The endogeneization of this rigidity is a desirable property since most macroeconomic models display ad hoc wage rigidity

[10] For more details on the Arrow-Borch-Wilson first-order condition on risk-sharing, see Rosen (1985). Ligon, Thomas and Worrall (2000) also discuss these risk-sharing first-order conditions.

(Rosen 1985). The insurance provided by the contract dissociates the behaviour of the real wage from that of the underlying marginal product of labour. It reads

$$w = mpl + TR$$

where *TR* stands for the transfer from the entrepreneur to the worker. These transfers may be allowances or penalties depending on the business conditions. They reduce the fluctuations of *w*. For instance, when business conditions are favourable, *mpl* is high and *TR* takes a negative value. The worker must then pay a penalty to the entrepreneur. In such case, $w < mpl$. Conversely, when business conditions deteriorate, *mpl* is low and *TR* is positive. In this scenario, the worker receives a positive transfer from the entrepreneur so $w > mpl$. This insurance device thus stabilizes *w*. Furthermore, *w* varies according to a step function when the variations in productivity are large enough.

In their 1988 paper, Thomas and Worrall focus on the implications of self-enforcing labour contract on the one period ahead dynamics of wages. Their main contribution is to show that the fixed end points of the wage interval imply a rigidity of the response of current wages to past events. In other words, their self-enforcing labour contract model delivers wage persistence, a theoretical prediction later successfully tested by Beaudry and DiNardo (1991). In their empirical work, Beaudry and DiNardo (1991) found that real wage follows a ratchet-like process, rising when the labour market is tighter but remaining constant otherwise[11]. Furthermore, the current wage is determined by the tightest labour market conditions incurred during the worker's tenure. Although the result of Beaudry and DiNardo tends to support the relevance of the self-enforcing contracts theory, Devereux and Hart (2007) argue that this result may be also compatible with a model of job-based wages and intermittent procyclical promotion, a variant of the spot market model obviously at odds with the analysis of Beaudry and DiNardo. However, the authors also acknowledge the fact the two views are observationally equivalent - i.e. that the self-enforcing labour market theory cannot be ruled out on that ground: "as such, the cyclical implications of the insurance contract model are observationally equivalent to the implications of the model with job-based wages and intermittent procyclical promotion."(Devereux and Hart 2007, p. 664).

[11] This downward rigidity stems from a specific feature of their model. By construction, it obtains from the one-sided limited commitment assumption in their model – the principal can fully commit to the relationship there.

As Calmès (1999), the 2007 article of Thomas and Worrall focuses on macroeconomic dynamics. Here the model aims at rationalizing some stylized facts regarding macroeconomic dynamics: the weak correlation between real wage and productivity; the high correlation between hours worked and productivity and the negative correlation between hours worked and wages. They also study the time path of consumption and hours worked using Frisch-type demand functions.

Compared to Thomas and Worrall (1988), and as in Calmès (2007), Thomas and Worrall (2007) introduce variable hours and a stochastic production function. At the start of each period, the entrepreneur and the worker observe the current state s_t. Any agent can quit and take his outside option. Otherwise, agents trade at the agreed terms, in which case the output is realized and the entrepreneur offers the contractual wage payment. The value (discounted utility) of the outside option of the worker and the firm, respectively, is denoted by $\chi_w(s)$ and $\chi_f(s)$ in state s. Contrary to Thomas and Worrall (1988), here the duration of the contract, denoted by T, is finite and random. At date $t = T$, after observing the current state s_t, the partnership dissolves and both agents get their outside option. T being a random variable, or a stopping time, the length of the contract will in general depends on the history of shocks. Note that in their 1988 article, Thomas and Worrall assumed an infinite horizon to account for the Holmström backloading principle. However, infinite horizon might not be necessary nor sufficient to avoid a contract set reduced to the flexible wage trivial singleton (Calmès 2007). Another way to deal with the backloading is to impose some restrictions on the risk-sharing domain. For example, Calmès (2007) proposes imperfect contract competition to deal with this matter. Thomas and Worrall (2007) develop a similar idea. They assume the existence of some reneging costs C_f and C_w (where f stands for the firm and w the worker).

Let $U_t(h_t)$ denote the continuation utility which the risk-averse worker gets from the contract from t onwards (assuming no termination at time t). This equation reads

$$U_t(h_t) = u(w_t(h_t), H(h_t)) + E\left[\sum_{\tau=t+1}^{T-1} \beta^{\tau-t} u(w_\tau(h_\tau), H(h_t)) + \beta^{T-t}\chi_w(s_T) \,\middle|\, h_t\right]$$

where $H(.)$ denotes the hours worked, and β the discount factor common to both parties. The other variables are the same as previously defined.

The risk-neutral entrepreneur's continuation profit is given by the following equation

$$\Pi_t(h_t) = z(s_t)H(h_t) - w_t(h_t)H(h_t) + E\left[\sum_{\tau=t+1}^{T-1} \beta^{\tau-t}(z(s_\tau)H(h_\tau) - w_\tau(h_\tau)H(h_\tau)) + \beta^{T-t}\chi_f(s_T) \middle| h_t\right]$$

It is assumed again that the worker consumes all his earnings so that

$$c(h_t) = w(h_t)H(h_t), \quad \forall t$$

with *c(.)* the consumption function. We can thus incorporate the consumption function into this equation and rewrite

$$\Pi_t(h_t) = z(s_t)H(h_t) - c(h_t) + E\left[\sum_{\tau=t+1}^{T-1} \beta^{\tau-t}(z(s_\tau)H(h_\tau) - c_\tau(h_\tau)) + \beta^{T-t}\chi_f(s_T) \middle| h_t\right]$$

The contract is self-enforcing if the following equations hold for all dates t, $T-1 \geq t \geq 1$, and for all h_t:

$$U_t(h_t) \geq \chi_w(s_t) - C_w$$

$$\Pi_t(h_t) \geq \chi_f(s_t) - C_f$$

with C_w and C_f being respectively the costs of reneging for the worker and the firm. They may be viewed as mobility costs for these agents. As previously mentioned, these costs indirectly and implicitly account for labour market imperfection, a property required for the self-enforcing contract labour market theory not to deliver a trivial allocation – for its absence would get us back to the flexible wage model allocation − cf Calmès 2003. These cost equations state that the contract must provide to the worker and the entrepreneur at least what they can get by quitting, net of quitting costs.

The presence of reneging costs is not innocuous. They are required both for the existence of a non trivial contract and for the sustainability of a limited-commitment partial equilibrium. Without these reneging costs, the surplus related to the spot market would be always greater than the one related to the contract, so no non trivial contract would be possible. Actually, these costs can be interpreted as down payments made in the first period (i.e. the core of the Holmström backloading principle previously mentioned). They are the equivalent of the imperfect contract competition parameter proposed in Calmès (2007).

The contracts specify the level of consumption c_t and the hours worked H_t. The resulting program, formulated with the variational location method reads as follows

$$\sup_{(c_t(h_t),H_t(h_t))_{t=1}^T} \Pi(h)$$

s.t.

the two SEC

$$U(h) \geq \chi_w(s_t) - C_w$$

$$\Pi(h) \geq \chi_f(s_t) - C_f$$

and the participation constraint of the worker

$$U(\delta;h) \geq \overline{U}$$

To find the time path of real wage and hours worked in their model, Thomas and Worrall (2007) first show that the condition for an efficient intertemporal allocation of hours holds in this model, so

$$-\frac{u_H(c_t(h_t),H_t(h_t))}{u_c(c_t(h_t),H_t(h_t))} = z(s_t) \qquad (1)$$

Hence, to have efficiency, the marginal rate of substitution between hours and consumption must equal the marginal product of labour $z(s_t)$[12]. Since the Lagrange multiplier λ is equal to the marginal utility of consumption

$$\lambda = u_c(c_t(h_t),H_t(h_t)) \qquad (2)$$

Equation (1) may thus be rewritten as

$$-u_H(c,H) = \lambda z \qquad (3)$$

From equations (2) and (3), we derive the Frisch-type demand functions $c(\lambda,z)$ and $H(\lambda,z)$. According to Kim (1993), these are obtained by holding the marginal utility of

[12] See Beaudry and DiNardo (1991). Note that in Thomas and Worrall (2007), the production function has a very simple formulation: $f = z(s_t)H(s_t)$. The marginal product of labour − the derivative of f with respect to $H(s_t)$ − is thus equal to $\frac{\partial f(s_t)}{\partial H(s_t)} = z(s_t)$.

wealth constant, so they are also called "marginal utility of wealth constant demand functions"[13]. These equations generally take the form: *f(λ,p), z* replacing the price effect in our two Frisch-type equations. In the standard RBC model, *w* (wage)[14] can be substituted for *z* since *w* generally equals *z* in this kind of model. Note also that in this kind of function, a variation of *z*, holding *λ* constant, gives way to a pure substitution effect, while a variation of *λ*, holding *z* constant, is equivalent to a pure income effect. Moreover, in these Frisch equations, *λ*, being also a shadow price, incorporates all the future information relevant for the consumption decision. In a standard RBC model, allowing borrowing and lending, equation (3) is replaced by

$$\lambda_t = (1 + r_t)\beta E(\lambda_{t+1}) \qquad (4)$$

This Euler equation shows directly how λ_t includes future information. Using equation (4) instead of equation (3) has important consequences for the computation of the time paths of wage and hours as well as the implied macroeconomic dynamics.

To link our Frisch-type functions *c*(.) and *H*(.) to wage, we must resort to one constraint of the model stating that all revenues are consumed, so the consumers neither save nor borrow

$$c_t(h_t) = \omega_t(h_t)H_t(h_t) \qquad (5)$$

To feature the time paths of wage and hours – following a technological shock *z* or a variation in *λ* – we adopt the Thomas and Worrall's (2007) methodology. In addition, because of the crucial role they play in the overall macroeconomic dynamics, we also examine the comovements between consumption and hours. Our focus here is thus different from that of Thomas and Worrall (2007).

First, we determine the sign of the derivatives of the two Frisch-type functions in the context of equation (3) where capital accumulation is excluded. For the hours function, the two derivatives H_z and H_λ are both positive. Thomas and Worrall (2007) give the following justification to these signs. A positive technological shock − i.e. an increase in *z* − holding *λ* constant, corresponds to a pure substitution effect, so $H_z > 0$. Furthermore, an increase in *λ* maintaining *z* fixed, that is holding the marginal rate of substitution constant,

[13] For more details on the derivation of this kind of functions, see Browning et al (1985).
[14] We remind that we use *w* to designate wage in a RBC model and *ω* to designate it in a limited-commitment model.

gives rise to a pure positive income effect which impacts positively on hours worked, leisure being assumed a normal good. H_λ is therefore also positive.

Let us now consider the signs of the derivatives of the consumption function with respect to the same arguments. An increase in λ entails a reduction in consumption, so C_λ is negative. In the case at hand (no savings), Thomas and Worrall (2007) show that the sign of C_z is ambiguous. However, we know that we have a pure substitution effect here, i.e. a compensated movement of z. If $u_{cH} = 0$, which corresponds to a separable utility function, there is no substitution effect at all, so $c_z = 0$. In other words, z impacts only on hours.

Second, we consider the paths of hours and wages in this limited-commitment model environment, still reasoning with the occurrence of a positive technological shock, i.e. an increase of z. Since $H_z > 0$, hours are procyclical. However, assuming separable consumers preferences, c_z is equal to 0 and equation (5) implies that ω decreases. Hours worked and wage are thus negatively correlated in this economy. This is the reason why the limited-commitment economy provides a correlation more in line with the stylized facts than the correlation implied by a classical model. Indeed, in Calmès (2007), hours worked are strongly correlated to productivity, while wage is not.

Thomas and Worrall (2007) did not study the comovement between consumption and hours worked in the framework of their model. Given our general argument about the importance of these overall comovements for understanding macroeconomic dynamics, we have to expose how they relate to the Frisch decomposition at hand. Following a positive technological shock (z), this comovement depends on the degree of separability of consumers preferences. If preferences are assumed separable − i.e $u_{cH} = 0$ − we have $H_z > 0$ and $c_z = 0$. Hence, holding λ constant, there is no comovement between H and c. In Calmès (2007), preferences being separable, the substitution effect is minimal. In the partial risk sharing economy, there is some income effect at play (i.e. $\lambda \neq 0$) so that a comovement between H and c is observed. However, with non-separable preferences, this comovement would be even more pronounced. Indeed, if $u_{cH} > 0$, we have $H_z > 0$ and $c_z > 0$. (Rosen 1985). Hence, in the limited-commitment economy, the comovement of these two variables is very dependent on the assumption regarding the separability of preferences, as this assumption directly influences the way the income effect and the substitution effect interact with one another – as in the classical economy, but only in a different manner.

As a matter of fact, to understand where this difference comes from, it is instructive to remind how consumption and hours worked commove in a standard RBC model. In this case, the Frisch-type decomposition derives from equation (4) where z can be replaced by w in the two Frisch-type demand functions. If preferences are separable ($u_{cH} = 0$), it is easy to show that $C_w = 0$ and $H_w > 0$. Hence, under these assumptions, there is no comovement between consumption and hours worked, which is exactly the same result as the one derived for the limited-commitment economy reported in the previous paragraphs. If preferences are non-separable with $u_{cH} > 0$, we then have $C_w > 0$, and the comovement is thus positive between consumption and hours worked as in the limited-commitment economy. However, as justifiedly argued in Thomas and Worrall (2007), we would have to assume that λ is fixed to establish these "equivalence results", a misleading assumption considering the law of motion λ_t inherits from the intertemporal substitution hypothesis. Indeed, in the standard case, the comovement between hours worked and consumption also depends on the evolution of λ. which represents the income effect of our Frisch-type demand functions, a much more active effect in the standard RBC framework.

3.2 Calmès Model (1999, 2003, 2007) and Macroeconomic Dynamics

Before Calmès (1999), macroeconomists who used self-enforcing labour contracts were not concerned by the correlation between the labour market variables and the dynamic propagation of macroeconomic shocks. They only dealt with the contemporaneous dynamics of the macroeconomic aggregates, not with their intertemporal dynamics − e.g. Boldrin and Horvath 1995. The first study which discusses the implications of self-enforceability of contracts on the transitory dynamics of labour market variables and its consequences on other key macroeconomic variables is Calmès (1999).

Calmès (1999, 2003) aims at investigating whether the real wage stickiness implied by the risk-sharing hypothesis can help explain macroeconomic stylized facts that the standard models have difficulty to rationalize. Calmès (2007) explicitely derives the potential effects of risk-sharing on the dynamics of employment, aggregate consumption and aggregate production.

In Calmès (2007), there are two interrelated principles which drive the transitory dynamics of the self-enforcing labour contracts. First, introducing an endogenous real rigidity in the model gives rise to an amplification of technological shocks. Wage stickiness is an artefact of the risk-sharing hypothesis. Second, Calmès (2007) conjectures that macroeconomic dynamics is partly driven by the weakening of the income effect inherent from a limited commitment problem. Indeed, following a positive shock, say a positive productivity shock, the substitution effect leads to an increase of aggregate employment and consumption but in standard models, a very strong income effect comes to dampen this dynamics. As a result, the shock is neither amplified nor propagated. Calmès (2007) shows that is not the case in a self-enforcing economy because the partial risk-sharing associated to limited-commitment weakens this income effect, freeing the substitution effect and hence fostering the fluctuations. We come full circle as the two principles relate to one another. The real rigidities, endogenized with self-enforcing labour contracts, generate a rich dynamics absent from RBC models. As a matter of fact, in a recent survey on RBC models, Rebelo (2005) noted that Wen (1998) model is one of the few RBC models attempting to match the properties of the spectral density of some important macroeconomic aggregates. To improve the performance of RBC models in terms of macroeconomic dynamics, Wen (1998) resorts to nonseparable preferences and to an employment externality giving way to an aggregate production function with increasing returns to scale. According to Wen (1998, p. 200), nonseparable preferences help propagate shocks while the employment externality helps amplify the shocks. Even if Wen (1998) model too fails to match the empirical spectral density of consumption, a key macroeconomic variable, it is the first to partly succeed in matching the spectral densities of output, investment and employment.

Calmès (2007) model features two important dimensions: imperfect contract competition and the initial bargaining power of the agents. One of his main contribution is to show that these two aspects are important factors influencing the transitory dynamics of the model.

Calmès (2007) model features three building blocks:

i) The spot market, represented by the flexible wage model where the income effect plays in full;

ii) The full-commitment model, a standard implicit contract model where there is no participation constraint. In this environment, the income effect is completely eliminated, with full insurance of earnings provided by the entrepreneur to the worker;

iii) The limited-commitment model itself, where the income effect, although present, is weakened. The macroeconomic dynamics derived from the model is a mixture of the one associated to the benchmark models (the spot market economy and the full-commitment one).

The features of the limited-commitment model are the following. Compared to Thomas and Worrall, Calmès introduces variable hours (*H*). The worker's utility function is separable. As in Thomas and Worrall, the worker does not save and consumes all his earnings so that $c(h_j) = w(h_j)H(h_j)$. His participation constraint is given by:

$$\forall \tau, \quad E_\tau \sum_{j=\tau}^{\infty} \beta^j u[c(h_j); H(h_j)] \geq E_\tau \sum_{j=\tau}^{\infty} \beta^j u[c^s(\varepsilon_j); H^s(\varepsilon_j)]$$

where the superscripts *s* denote spot market variables, the outside opportunity being represented by the spot market.

The entrepreneur operates a production function taking the general form $F(\varepsilon_j, H(h_j), k)$. An important innovation in Calmès (2007) with respect to Thomas and Worral (1988) setting is the introduction of a production function whose arguments are hours and capital, this last input being absent in the former limited-commitment models. This is part of his effort to study the Thomas and Worrall theory in a macroeconomic environment. However, at this stage, *k* is held constant, for tractability. In his model, uncertainty comes from an exogenous technological shock, denoted ε_t,, a state variable that can take *N* different values each period $\{\varepsilon_t^1, ..., \varepsilon_t^N\}$. p_{ij} represents the transition probability to be in state ε^j conditional on the previous state ε^i, $\forall i = 1,...N, \quad \forall j = 1,...N$. For more tractability, p_{ij} is assumed to be equal to 1/N.

The profit function of the firm, defined for a given history of states (h_j), is:

$$F(\varepsilon_j, k, H(h_j)) - w(h_j)H(h_j) - r(h_j)k, \ \forall j$$

where r is the marginal product of capital. $r(h_j)k$ is thus the cost of capital for a given h_j. We can resort to this profit function to write the participation constraint of the entrepreneur

$$\forall \tau, \quad \sum_{j=\tau}^{\infty} \beta^j \{F(z_j; k; n(h_j)) - w(h_j)n(h_j) - r(h_j)k\} \geq 0$$

The self-enforcing contract must offer the entrepreneur at least zero profit, a level consistent with the average outcome of perfect competition assumed in the flexible wage environment, i.e. the spot market economy.

With the introduction of capital in the model, the external opportunity relates explicitly to business conditions. Adding capital in the production function makes the scenario where profit is zero an extreme (limit) case. In general, with the help of capital, the gains of the entrepreneur will be positive and will be conditional on the state of business conditions, especially the level of productivity, because the opportunity cost of capital is a function of the interest rate, which, in turn, depends on business conditions. This contribution of Calmès (2007) was underlined by Thomas and Worrall (2007). They note that, besides proposing one model where hours worked are assumed variable – an essential generalization if we aim at adapting the theory of self-enforcing contracts to the study of macroeconomic fluctuations – Calmès (2007) also introduces an endogenous outside opportunity which really depends on business conditions.

Although a dispensable hypothesis, for simplicity, the duration of a contract is assumed infinite in the model. A history dependent contract

$$\delta_{h_j} = \{w(h_j); H(h_j)\}_{j=1}^{\infty}$$

specifies a wage and labour input pair for every realization of nature. In the limited-commitment model, a contract is self-enforcing if no party wants to renege on the relationship.

In addition to infinite duration, some frictions are introduced in the models to avoid the degenerate result of a quasi-empty contract set. Indeed, perfect contract competition would drive the allocations toward their first best levels, where no level of risk-sharing is sustainable, and only the flexible wage equilibrium can exist.

For this matter, Calmès (2007) introduces an exogenous parameter P accounting for imperfect contract competition, $P = 1$ being the maximum value, the one associated to the pure contract competition that would prevail in a perfectly flexible wage economy. In this case, the spot market allocation associated to the perfect contract competition provides the maximum feasible surplus to the worker. In other words, whenever $P \neq 1$, this autarkic equilibrium, the classical first-best, cannot be reached. To each contract belonging to the contract set is also associated a unique level of initial bargaining power distribution. This bargaining power is related to the utility level expected by the worker − an endogenous state variable.

In Calmès (2007), the dynamic program computing the Pareto efficient contract set associated to the limited-commitment model may be formulated as

$$\Pi(\varepsilon_s, U) = \sup_{w,H,\{U_j\}_j} F(\varepsilon_s, k, H) - wH - r(\varepsilon_s)k + \beta \sum_{j=1}^{N} \Pi(\varepsilon_j, U_j) p_{sj}$$

s.t.

the self-enforcing constraint of the firm

$$\Pi(\varepsilon_j, U_j) \geq 0 \quad \forall j = 1, \ldots, N$$

the self-enforcing constraint of the worker

$$U_j \geq V^s[\varepsilon_j] \quad \forall j = 1, \ldots, N$$

the budget constraint of the worker[15]

$$c = wH$$

[15] It binds at equilibrium.

and the participation constraint related to the expected utility of the worker

$$u(wH,H)+\beta\sum_{j=1}^{N}U_j p_{sj}\geq U$$

In this program, p_{sj} represents the transition probability to be in state ε^j knowing that the previous state was ε^s, and U is the promised utility level chosen in the $\{U_j\}_j$ menu given the realization ε^j. The self-enforcing constraint of the worker states that, each period, the entrepreneur must promise at least this level U from the contingent menu $\{U_j\}_{j=1}^N$. This is a major contribution of Thomas and Worrall since it deals directly with the usual problem of the history dependence of the equilibrium allocations[16].

The full-commitment model, i.e. one benchmark model[17], is the limited-commitment model without these two self-enforcing constraints. The dynamic program corresponding to this model is the following:

$$\Pi^*(\varepsilon_s,U)=\sup_{w,H,U} F(\varepsilon_s,k,H)-wH-r(\varepsilon_s)\,k+\beta\sum_{j=1}^{N}\Pi^*(\varepsilon_j,U_j)\,p_{sj}$$

s.t.

the binding constraint of the worker

$$c = wH$$

and the participation constraint related to the expected utility of the worker

[16] On that matter, see: Ljungqvist and Sargent (2004), chap. 19.

[17] One of the most important contributions of Thomas and Worrall (1988) is to show that the limited commitment allocation can obtain with the first iteration guess set to the full commitment value function.

$$u(wH,H)+\beta\sum_{j=1}^{N}U_j p_{sj}\geq U$$

We must now examine how the degree of contract competition and the bargaining power of the worker are introduced in the dynamic programs.

Let us begin by the parameter $P\in(0,1)$, which represents the degree of contract competition. In the dynamic program associated to the limited-commitment model, this parameter is used to compute the expected utility level of the outside option V^s. This expected utility represents the external opportunity of the worker he would enjoy if he worked on the spot market. V^s is equal to:

$$V^s = PV^s_{\max}$$

For instance, if imperfect contract competition is associated with a 10% welfare loss, *P* is then equal to 0.9 and $V^s = 0.9V^s_{\max}$.

Importantly, $V^s_{\max}$ is the value function associated to the maximum feasible surplus under autarky. The corresponding spot market economy features a single self-employed agent working in a frictionless environment. For this reason, $V^s_{\max}$ is the first best value only feasible if contracts were perfectly substitutes and wages purely flexible. In other words, this representation of the outside economy is one of perfect competition. In our approach, we rule out the possibility that wages can be fully flexible by assuming $P\neq 1$. Because of this imperfect contract competition, there is room for risk-sharing.

$V^s_{\max}$ is equal to

$$V^s_{\max}(\varepsilon_s)=\sup_{\{H^s_{\max}\}} u\left(F\left(\varepsilon_s,k,H^s_{\max}\right)-r(\varepsilon_s)k;H^{\max}_s\right)+\beta\sum_{j=1}^{N}V^s_{\max}(\varepsilon_j)p_{sj} \quad (6)$$

There is a third (implicit) agent in the limited-commitment model: the creditor who provides funds to finance the acquisition of capital[18]. Hence the presence of $r(\varepsilon_s)k$ in equation (6). The financial intermediary rents the capital to the firm.

While the degree of contract competition is exogenous in Calmès (2007) model, the degree of bargaining power of the worker is not: as in Thomas and Worrall (1988), it is

[18] A full formalization of this agent is still to be investigated.

endogenous and recorded with the promised utility menu. In other words, in the dynamic program, the menu function U determines the endogenous bargaining power of the worker. The worker's full bargaining power, U^{max}, is associated to a zero profit for the manager of the firm. It is obtained implicitly by solving the following equation

$$\Pi\left(\varepsilon_j, U^{max}\left(\varepsilon_j\right); U_j\right) = 0, \forall j = 1, \ldots, N$$

Under limited-commitment, consumption is bounded by the self-enforcing constraint of the worker. The upper bound of the admissible consumption interval corresponds to the consumption associated with the worker's full bargaining power (i.e. U_{MAX}). The lower bound is reached when the entrepreneur has full bargaining power.

Since Boldrin and Horvath (1995) were the first to consider the macroeconomic implications of self-enforcing contracts, to implement his model, Calmès (2007) calibrates the utility function of the worker and the production function of the entrepreneur using the same kind of functions as these authors[19]. The utility function of the worker is a time-separable Bernouilli utility function of the CES class given by

$$\forall t, \quad u\left(c_t; H_t\right) = \frac{1}{1-\gamma} c_t^{1-\gamma} + \frac{\theta}{1-\gamma}\left(T - H_t\right)^{1-\gamma}, \ \gamma \in \left(0,1\right)$$

where T represents the total of non-sleeping hours and γ captures the risk aversion of the worker. The production function of the entrepreneur is a standard Cobb-Douglas production function, multiplicative in the shock variable ε_t. It has the following expression:

$$\forall t, \quad F\left(\varepsilon_t; k; H_t\right) = \varepsilon_t k^{\alpha} H_t^{1-\alpha}$$

As already mentioned, the introduction of capital inside the production function represents a specific novelty of Calmès (2007): in other well-known limited-commitment models, the production function depends only on hours worked and technological shock. This contribution is crucial as it implies a state dependent outside opportunity. Referring to Calmès (2007) model, Thomas and Worrall (2007) note that in much of the existing literature on limited-commitment labour contracts, it is assumed that competition among firms drives profits to zero − in which case the flexible wage allocation is the only one to

[19] However, as previously stated, Boldrin and Horvath (1995) consider one period contracts to investigate the contemporaneous dynamics of the macroeconomic aggregates. By contrast, Calmès (2007) is interested by the *intertemporal* macroeconomic dynamics implied by the Thomas and Worrall theory.

survive. This is no longer the case here, as firms must cover their capital costs and profits only need to be zero, on average, at equilibrium. The marginal product of capital, denoted *r*, is a state-dependent variable in the Calmès (2007) model. The outside opportunity of the firm is therefore state dependent and related to the level of *r*, which yields profit fluctuations from one state to another.

Before discussing the dynamics of the Calmès (2007) limited-commitment model, it is useful to first compare the dynamics of the two limit cases chosen as benchmarks: our flexible wage model, and the full-commitment economy. We explained previously that the dynamics is related to the relative strength of the substitution and income effects in these two models.

The flexible wage model environment (spot market) and the full-commitment economy display very different dynamic properties. Because the income effect has a strong influence in the spot market economy, the model of autarky displays weak dynamics, no amplification and no persistence. As conjectured in Calmès (1999), the income effect prevents the substitution effect to fully play its role, hence dampening the fluctuations (a limitation common to most RBC models). On the spot market, following a technological shock ε_t, consumption and hours worked return to their steady state values very quickly at the end of the first period. Furthermore, the time path of these variables is almost flat. In this respect, the flexible wage model is definitively not a suitable setup to account for the macroeconomic dynamics stylized facts.

The dynamics of the full-commitment model, a building block of the general model, is substantially different from that of the flexible wage one. Here, the risk-sharing is perfect, its domain unbounded, and there is a perfect smoothing of consumption across time and states[20]. Since he is perfectly insured, the worker does not have to bother about income uncertainty. Indeed, in this case, there is no income effect at all as it is eliminated with the risk-sharing hypothesis. Hence, in the full-commitment model, the propagation mechanism is only driven by the substitution effect, without impediment. As a matter of fact, when risk-sharing is perfect, initial shocks have an infinitely permanent effect on the

[20] With non-separable preferences, a perfect smoothing of marginal utility of consumption will still obtain (Thomas and Worrall 1988).

economy, and the shocks amplification and persistence are much larger than in the classical environment where both are actually absent.

The limited commitment economy is an hybrid situation where the dynamics of the two building blocks previously described intertwine. In the limited-commitment model, the income effect is not totally removed but it is diminished by the presence of the partial risk-sharing. The insurance provided by the entrepreneur smoothes partially consumption across time and states and thus weakens the income effect. In itself, the degree of consumption smoothness is conditional on the level of the initial distribution of bargaining power, on the dynamics of the bargaining power of the worker, and on the degree of imperfect contract competition. As noted previously, these are central contributions of Calmès (2007) and very important to understand how a limited-commitment economy can help explain macroeconomic dynamics.

Following any shocks, even a purely transitory one and before arriving at its steady state level, the economy displays a phase of transitory dynamics which crucially depends on the initial bargaining power distribution. One of the key contributions of Calmès (2007) is to show that the initial distribution of bargaining power plays a crucial role in explaining the transitory macroeconomic dynamics[21]. For instance, if the initial bargaining power of the worker is very low relative to that of the employer, the fluctuations of the economy will be larger in this transitory phase[22]. Then incoming shocks have a prolonged impact recorded in *U*. As bargaining power is set higher, with more balanced distribution of its initial value, these fluctuations are smaller because the perfect risk-sharing domain is reached more rapidly. The worker maximum bargaining power is obtained when the entrepreneur's profit is reduced to zero. By varying this initial value, the whole contract set obtains.

Another original finding of Calmès (2007) relates to the impact of the level of *P*, i.e. the degree of contract competition imperfection, on economic fluctuations. As *P* increases,

[21] Although the consecutive distribution of bargaining powers is an endogenous state variable of the model, once the relationship starts, Calmès (2007) does not make any particular assumption regarding the initial bargaining power of the worker.

[22] The same is true if the employer has a high initial bargaining power. When the initial bargaining power is evenly distributed, Calmès (2007) explains that the dynamics is similar to the one obtained under full-commitment.

the shocks have more likely an initial impact on the economy but the amplitude of this impact is smaller[23].

Calmès (2007) also confirms Beaudry and DiNardo (1991) findings on how aggregate dynamics crucially depends on the transitory dynamics observed at the beginning of the contract. Calmès (2007) finds that this transitory dynamics is conditioned by the initial bargaining power of the worker and by the degree of imperfection in contract competition. Although not formalized per se, this distribution of the initial bargaining powers between the worker and the entrepreneur is trivially dependent on the initial business conditions, e.g. the level of capital and its marginal productivity, two new dimensions introduced in this limited-commitment framework.

In summary, Calmès (2007) studies the intertemporal macroeconomic dynamics of a limited-commitment economy, a research avenue unexplored in previous studies dealing with self-enforcing contracts. The findings are very encouraging since they suggest that it is possible to generate a rich macroeconomic dynamics when adopting the self-enforcing contracts theory of Thomas and Worrall.

4. Macroeconomic Dynamics and the Theory of Self-Enforcing Labour Contracts: Some Research Avenues

4.1 The Utility Function of the Worker

Risk-sharing is an important motive for contractual solutions. As originally shown by Calmès (1999, 2003 and 2007), it provides insurance to workers who can thus adopt more opportunistic behaviour vis-à-vis business conditions, so that the income effect, which generally dampens the response of the economy to exogenous technological shocks, comes into play later and with diminished strength leaving more room to the substitution effect.

In this respect, one interesting topic to investigate is the role played by preferences in this kind of models, and more precisely, to study how the dynamic properties of the model change with the formulation of the individual utility function. Indeed, for tractability,

[23] It would be interesting to study to what extent the "great moderation phenomenon" could relate to the institutional innovations on the labour market.

preferences of the worker are assumed to be separable in consumption and leisure in Calmès (2007). However, resorting to this kind of preferences reduce the overall comovements between hours and consumption since, in this case, the two variables tend to be less complementary – ceteris paribus. The modelled comovements are thus lowered because of the increased power the income effect inherits from this kind of preferences. Hence, it will be useful to consider the more general case of non-separable utility functions of the RBC type[24]. In this respect, it is worth noting that recently, Jaimovich and Rebelo (2006) have developed a general formulation of nonseparable preferences in consumption (C_t) and hours worked (H_t) given by the following equation:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \frac{\left(C_t - \psi\, H_t^{\theta} X_t\right)^{1-\sigma} - 1}{1-\sigma}$$

where

$$X_t = C_t^{\gamma} X_{t-1}^{1-\gamma}$$

The presence of X_t implies that, in general, preferences are time nonseparable in consumption and hours worked. But depending on parametrization, the case of separable preferences can also apply here. According to Jaimovich and Rebelo (2006), the two classes of utility functions most widely used in the RBC literature are nested in this function. When $\gamma = 0$, the preferences proposed by Greenwood, Hercowitz and Huffman (1988) obtain, and when $\gamma = 1$, the famous King, Plosser and Rebelo[25] (1988a) preferences are used.

The utility function proposed by King et al. (1988) becomes separable when expressed in logarithmic form but, in level, the Greenwood et al. (1988) preferences, by construction, completely eliminate the income effect, with the disadvantage however that, compared to the King et al. (1988 a,b) case, as a consequence, the economy cannot evolve on a balanced growth path.

In a self-enforcing contract, the income effect being weakened by risk-sharing, it might be conjectured that relying on nonseparable preferences (i.e. with $\gamma \to 0$) could dampen even more this effect, what could, in turn, strengthen the propagation mechanism generated by limited-commitment. On the other hand, there is a possible arbitrage to face, because, at least

[24] As a matter of fact, Calmès (2007) suggests to study the use of nonseparable utility functions.
[25] Their formulation of preferences is standard as it is compatible with a balanced growth path.

in the one-sided limited-commitment model presented by Cahuc and Zylberberg (2001), preferences must be separable to make the wage sticky, i.e. insensitive to a technological shock, which then leads to elimination of the income effect. Since the income effect hinders macroeconomic dynamics, the effect of the degree of preferences separability on macroeconomic dynamics is not clear and thus deserves further research.

4.2 Variable Capital and Self-Enforcing Contracts

As Rosen (1985) has quite rightly pointed out, capital allows the aggregate disturbance to be partially diversified, through capital accumulation in favourable aggregate conditions and decumulation in unfavourable circumstances. This classical intertemporal trade reduces the income effect of aggregate shocks on consumption and employment and accentuates pure substitution effects. According to Calmès' (1999) conjecture, this, in turn, works as a propagation mechanism different from the one generated by limited-commitment. For researchers interested by data matching experiments, introducing variable capital in the theoretical framework of limited-commitment may help replicate the macroeconomic dynamics stylized facts better. In this respect, Calmès (2007) innovates by introducing capital in the production function of his model. But there, capital is held constant for tractability. Actually, making capital variable leads to some complications. The distribution of capital holdings across entrepreneurs becomes part of the state. The distribution of workers across entrepreneurs will depend on this distribution of capital holdings and the equilibrium will thus be conditional on the distribution of capital holdings.

Moreover adding these kinds of interactions in a self-enforcing labour contract model entails a contract set which is not necessarily compact, which creates an additional difficulty for solving the model. However, to tackle this issue, a lottery could be used to convexify the contract set as in Ligon, Thomas and Worrall (2000). For that matter, the third agent (i.e. the financial intermediary) could also be explicitly introduced and a second relationship added to the toy economy, for instance a bank linked to the firm.

4.3 The Question of the Aggregation of Contracts

The aggregation of heterogeneous individual contracts is another important question to be addressed. After having considered firms as homogeneous (our "representative firm" framework), it would be interesting to consider the case of heterogeneous firms. As noted previously, one of the principal results reported in Calmès (2007) is that aggregate macroeconomic dynamics depends crucially on the transitory dynamics observed at the beginning contracts. Transitory dynamics is, in turn, conditioned by the initial bargaining power of the worker (as by the degree of imperfection of contract competition). And the initial distribution of the relative bargaining power between the worker and the entrepreneur itself depends on the hiring conditions. To investigate aggregate dynamics in this context, it will thus be important to build an economy where contracts overlap. To do so, a technique aggregating heterogeneous contracts is needed. To aggregate such contracts, it is important to consider that the external opportunities of the agents should really depend on the state of the economy, especially on other contracts. In this respect, the question also relates to the endogeneization of imperfect contract competition.

4.4 The Contracts Duration

Regarding the duration of contracts, there are two extreme approaches in the limited-commitment literature. Some, like Boldrin and Horvath (1995), set the duration of contracts to one period. Obviously, the macroeconomic dynamics analyzed is thus horizon constrained in this case. Researchers can at most consider contemporaneous macroeconomic dynamics. To avoid the backloading result, most authors in the field of limited-commitment theory set the duration of contracts to infinity. There is a technical reason for this assumption. According to Thomas and Worrall (1988), if workers had only finite life, they would surely renege on any contract which pays below the spot market rate in the final period. The model will then converge to the trivial equilibrium associated to a pure and perfect contract competition. Hence, since the very beginning of a contract, the only sustainable allocation would be the one associated to the regular flexible wage economy.

Thomas and Worrall (1988) also assume that entrepreneurs will try to accommodate workers in order to maintain a good reputation or to be in a position to attract workers in the future. However, relying on infinite duration is a very strong assumption. For example, it prevents analysing such interesting topics as pension plans and retirement dates. Besides, the infinite horizon hypothesis is neither sufficient nor necessary to rule out the backloading result. Actually, as long as contract competition is perfect, this result obtains because the autarkic Pareto allocation dominates strictly. As explained earlier, another way to avoid the backloading result is to directly introduce imperfect contract competition. In this case, it is possible to relax the assumption of infinite duration and consider the case of random termination dates.

Let us notice that, insofar as we incorporate the assumption of a random date for the duration of a contract, we must account for a third kind of heterogeneity. The initial model, where heterogeneity is minimal, because of the presence of only two different representative agents, has to be generalized to deal not only with the distribution of the initial allocation of capital and the initial conditions under which firms begin their activities but also the life expectancy of each firm. This enlarges the states set exponentially, which might lead to the curse of dimensionality − bear in mind that we do not compute a fixed point but a fixed "interval", i.e. an infinity (continuum) of points.

4.5 Endogenizing the External Opportunities

We have previously exposed how Calmès (2007) endogenizes the external opportunities of the worker and of the manager. As noticed by Thomas and Worrall (2007), this is the right way to proceed because the transposition of the theory of self-enforcing contracts to macroeconomics must account for the more general case where the external opportunities depend on the whole past history of the contracts and on the rational expectations about their future histories.

In Thomas and Worrall (1988) article, which paved the way to all subsequent macroeconomic models featuring self-enforcing labour contracts, the external opportunities were represented by an ad hoc spot market. For the worker, the self-enforcing constraint ensures that he is not better off by taking advantage of an external opportunity, working forever on the spot market if he reneges. After Thomas and Worrall (1988), most of the following studies have tried to consider that the external opportunity of the worker was determined by the discounted expected utility he will get from being employed elsewhere in the economy. But usually, this external opportunity remains partly exogenous or imperfectly endogenous.

There are many ways to endogenize the external opportunities of the agents. One, just mentioned, would consist in relating it to business conditions of a contractual economy. Another is presented next. We consider that the external opportunity is in fact an option and resort to the theory of derivatives to create endogenous external opportunities.

4.6 Self-Enforcing Contract and the Option Theory

As noted earlier, the outside opportunities of the worker and of the entrepreneur are actually options. The insurance provided by the entrepreneur erases the left tail of the distribution of the worker's payoffs, so this distribution is truncated as it is the case for financial or real options. In the theory of options, insurance is provided by a put. For instance, the outside opportunity of the worker may be considered as a put whose exercise price corresponds to the value of his outside opportunity. This put sets a floor to the gains of the worker, this floor being precisely the value of his outside opportunity. The payoff[26] of such a put is thus

$$payoff = \left(U^s - U^c\right)^+$$

where U^s, the exercise price of the put, corresponds to the value of the outside opportunity of the worker at termination date, say a contract on the spot market, and U^c are the gains that the worker gets from the contract. Importantly U^s can be endogenized by considering that the outside opportunity of the worker is an exchange option. If the worker reneges, he will receive this payoff, that is he will cash his insurance.

Let us notice that the computation of the price of an option is perfectly compatible with the methodology of dynamic programming. Assume that *U(S(t),t)* is the value of an option at time *t*, *S(t)* being the price of its underlying asset. We can rely on the following Bellman equation to price this option (Tavella 2002)

$$U = sup\left\{F,\ \beta E(U')\right\}$$

[26] The payoff of an option is its value at maturity.

where F stands for the payoff of the option, which is its value upon exercise, the value of an option being the maximum of its value upon exercise (stopping value) and its continuation value $\beta E(U')$.

4.7 Self-enforcing labour market theory and growth

The question of economic growth can also be investigated in a limited-commitment economy. Most of the researchers in the field of self-enforcing labour contracts theory only consider a stationary economic framework. In such a framework, macroeconomic aggregates record no growth and the equilibrium converges towards a stationary or ergodic state. But it is well-known that most macroeconomic time series have a unit root or a deterministic trend, and in any case grow.

Once again, when the modelled economy is not stationary, we are confronted to the problem related to the non-convexity of the contract set. Should growth be studied in this kind of models, the link between stationarity and set convexity must thus be addressed. Indeed, since contracts depend on the past history of the firm, there exists a different contract for each level of capital and any endogenous state variable of the model. The objective is to arrive at the optimal contract resulting from a multitude of possible allocations.

To add growth to a limited-commitment model, a stochastic or a deterministic trend must be added to all macroeconomic variables depending on their nature. If the trends of the macroeconomic variables incorporated in the model are all deterministic, the model can be made stationary by rescaling these variables, i.e. by dividing them by their deterministic trend. But a macroeconomic variable which has a unit root, or equivalently which has a stochastic trend, cannot be "detrended" by following a procedure similar to the one used for variables having a deterministic trend. These variables must be expressed in first differences to make them stationary. Once transformed, a lottery must then again be applied to convexify the contract set solutions.

5. Conclusion

This article exposes how to think about the self-enforcing labour market theory to study macroeconomic dynamic stylized facts, especially transitory macroeconomic dynamics, most previous models in this field being only concerned with wage contract dynamic properties or contemporaneous dynamics.

Since the seminal paper of Thomas and Worrall (1988), who made an allusion to contemporaneous dynamics in their concluding remarks, there has been no comprehensive developments on that matter until the contribution of Calmès (1999, 2003, 2007). As explained in this chapter, his work opens the door to a very promising field of research.

At this stage, these developments are still in their infancy and this avenue remains an emerging topics in macroeconomics. The changes to the existing limited-commitment models which are required to have them match the macroeconomic dynamic stylized facts about transitory dynamics present a challenging task. As mentioned, many of the proposed changes imply non-convex contract sets, which creates an important difficulty for solving these kinds of models. Fortunately, there are ways to deal with it, and with the proposed modifications, the macroeconomic models of limited-commitment economies will feature an internal propagation mechanism of shocks much stronger than the one used in the RBC literature (i.e. capital accumulation) whose dynamics is quite weak, to say the least.

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